

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4723

Core Mathematics 3

MARK SCHEME

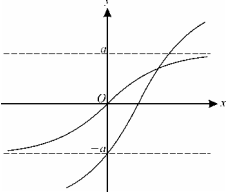
Specimen Paper

MAXIMUM MARK	72
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This mark scheme consists of 4 printed pages.

<p>1 <i>EITHER:</i> $4x^2 + 4x + 1 > x^2 - 2x + 1$ i.e. $3x^2 + 6x > 0$ So $x(x+2) > 0$ Hence $x < -2$ or $x > 0$</p> <p><i>OR:</i> Critical values where $2x+1 = \pm(x-1)$ i.e. where $x = -2$ and $x = 0$</p> <p>Hence $x < -2$ or $x > 0$</p>	<p>M1 A1 M1 A1 A1</p> <p>M1 B1 A1 M1 A1</p>	<p>For squaring both sides For reduction to correct quadratic For factorising, or equivalent For both critical values correct For completely correct solution set</p> <p>For considering both cases, or from graphs For the correct value -2 For the correct value 0 For any correct method for solution set using two critical values For completely correct solution set</p>
<p>2 (i) $\sin x(\frac{1}{2}\sqrt{3}) + \cos x(\frac{1}{2}) + (\sqrt{3})(\cos x(\frac{1}{2}\sqrt{3}) - \sin x(\frac{1}{2}))$ $= \frac{1}{2}\cos x + \frac{3}{2}\cos x = 2\cos x$, as required</p> <hr/> <p>(ii) $\sin 45^\circ + (\sqrt{3})\cos 45^\circ = 2\cos 15^\circ$ Hence $\cos 15^\circ = \frac{1+\sqrt{3}}{2\sqrt{2}}$</p>	<p>M1 A1 M1 A1</p> <p>M1 A1</p>	<p>For expanding both compound angles For completely correct expansion For using exact values of $\sin 30^\circ$ and $\cos 30^\circ$ For showing given answer correctly</p> <p>For letting $x = 15^\circ$ throughout For any correct exact form</p>
<p>3 (i) $x_2 = \sqrt[3]{7} = 1.9129\dots$ $x_3 = 1.9517\dots$, $x_4 = 1.9346\dots$ $\alpha = 1.94$ to 2dp</p> <hr/> <p>(ii) $x = \sqrt[3]{17-5x} \Rightarrow x^3 + 5x - 17 = 0$</p> <hr/> <p>(iii) <i>EITHER:</i> Graphs of $y = x^3$ and $y = 17 - 5x$ only cross once Hence there is only one real root</p> <p><i>OR:</i> $\frac{d}{dx}(x^3 + 5x - 17) = 3x^2 + 5 > 0$ Hence there is only one real root</p>	<p>B1 M1 A1</p> <p>M1 A1</p> <p>M1 A1</p>	<p>For 1.91... seen or implied For continuing the correct process For correct value reached, following x_5 and x_6 both 1.94 to 2dp</p> <p>For letting $x_n = x_{n+1} = x$ (or α) For correct equation stated</p> <p>For argument based on sketching a pair of graphs, or a sketch of the cubic by calculator For correct conclusion for a valid reason</p> <p>For consideration of the cubic's gradient For correct conclusion for a valid reason</p>
<p>4 (i) $\int_0^2 (4x+1)^{-\frac{1}{2}} dx = \left[\frac{1}{2}(4x+1)^{\frac{1}{2}} \right]_0^2 = \frac{1}{2}(3-1) = 1$</p> <hr/> <p>(ii) $\pi \int_0^2 \frac{1}{4x+1} dx = \pi \left[\frac{1}{4} \ln(4x+1) \right]_0^2 = \frac{1}{4} \pi \ln 9$</p>	<p>M1 A1 M1 A1</p> <p>M1 A1 M1 A1</p>	<p>For integral of the form $k(4x+1)^{\frac{1}{2}}$ For correct indefinite integral For correct use of limits For given answer correctly shown</p> <p>For integral of the form $k \ln(4x+1)$ For correct $\frac{1}{4} \ln(4x+1)$, with or without π Correct use of limits and π For correct (simplified) exact value</p>

<p>5</p> <p>(i) 200 °C</p> <hr/> <p>(ii) $150 = 200 - 180e^{-0.1t} \Rightarrow e^{-0.1t} = \frac{50}{180}$ Hence $-0.1t = \ln \frac{5}{18} \Rightarrow t = 12.8$</p> <hr/> <p>(iii) $\frac{d\theta}{dt} = 18e^{-0.1t}$ Hence rate is $18e^{-0.1 \times 12.8} = 5.0$ °C per minute</p>	<p>B1 1</p> <hr/> <p>M1 M1 A1 3</p> <hr/> <p>M1 A1 M1 A1 4</p> <p style="text-align: right;">8</p>	<p>For value 200</p> <hr/> <p>For isolating the exponential term For taking logs correctly For correct value 12.8 (minutes)</p> <hr/> <p>For differentiation attempt For correct derivative For using their value from (ii) in their θ For value 5.0(0)</p>
<p>6</p> <p>(i) Domain of f^{-1} is $x \geq 1$ Range is $x \geq 0$</p> <hr/> <p>(ii) If $y = 1 + \sqrt{x}$, then $x = (y-1)^2$ Hence $f^{-1}(x) = (x-1)^2$</p> <hr/> <p>(iii) The graphs intersect on the line $y = x$ Hence x satisfies $x = (x-1)^2$ i.e. $x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$ So $x = \frac{1}{2}(3 + \sqrt{5})$ as x must be greater than 1</p>	<p>B1 2</p> <hr/> <p>M1 A1 2</p> <hr/> <p>B1 B1 M1 A1 4</p> <p style="text-align: right;">8</p>	<p>For the correct set, in any notation Ditto</p> <hr/> <p>For changing the subject, or equivalent For correct expression in terms of x</p> <hr/> <p>For stating or using this fact For either $x = f(x)$ or $x = f^{-1}(x)$ For solving the relevant quadratic equation For showing the given answer fully</p>
<p>7</p> <p>(i) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$</p> <hr/> <p>(ii) $\frac{8t}{1-t^2} + 3 \times \frac{1}{t} \times (1+t^2) = 0$ Hence $8t^2 + 3(1-t^2)(1+t^2) = 0$ i.e. $3t^4 - 8t^2 - 3 = 0$, as required</p> <hr/> <p>(iii) $(3t^2 + 1)(t^2 - 3) = 0$ Hence $t = \pm\sqrt{3}$ So $x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$</p>	<p>B1 1</p> <hr/> <p>B1 B1 M1 A1 4</p> <hr/> <p>M1 A1 A1 A1 4</p> <p style="text-align: right;">9</p>	<p>For correct RHS stated</p> <hr/> <p>For $\cot x = \frac{1}{t}$ seen For $\sec^2 x = 1 + t^2$ seen For complete substitution in terms of t For showing given equation correctly</p> <hr/> <p>For factorising or other solution method For $t^2 = 3$ found correctly For any two correct angles For all four correct and no others</p>

<p>8 (i) $\frac{dy}{dx} = \frac{2 \ln x}{x}$</p> $\frac{d^2y}{dx^2} = \frac{x(2/x) - 2 \ln x}{x^2} = \frac{2 - 2 \ln x}{x^2}$ <hr/> <p>(ii) For maximum gradient, $2 - 2 \ln x = 0 \Rightarrow x = e$</p> <p>Hence P is $(e, 1)$</p> <p>The gradient at P is $\frac{2}{e}$</p> <p>Tangent at P is $y - 1 = \frac{2}{e}(x - e)$</p> <p>Hence, when $x = 0$, $y = -1$ as required</p>	<p>M1 A1 M1 A1</p> <p>M1 A1 A1√ A1√ M1 A1</p>	<p>For relevant attempt at the chain rule</p> <p>For correct result, in any form</p> <p>For relevant attempt at quotient rule</p> <p>4 For correct simplified answer</p> <hr/> <p>For equating second derivative to zero</p> <p>For correct value e</p> <p>For stating or using the y-coordinate</p> <p>For stating or using the gradient at P</p> <p>For forming the equation of the tangent</p> <p>6 For correct verification of $(0, -1)$</p> <p style="text-align: center;">10</p>									
<p>9 (i) $a = \frac{1}{2}\pi$</p> <hr/> <p>(ii) $x = \tan(\frac{1}{4}\pi) = 1$</p> <hr/> <p>(iii)</p>  <p style="margin-left: 300px;">Asymptotes are $y = \pm 2a$</p> <hr/> <table border="0" style="margin-left: 40px;"> <tr> <td>x</td> <td>$\tan^{-1} x$</td> <td>$2 \tan^{-1}(x-1)$</td> </tr> <tr> <td>1.535</td> <td>0.993</td> <td>0.983</td> </tr> <tr> <td>1.545</td> <td>0.996</td> <td>0.998</td> </tr> </table> <p>Hence graphs cross between 1.535 and 1.545</p> <hr/> <p>(v) Relevant values of $(\tan^{-1} x)^2$ are (approximately) $0, 0.0600, 0.2150, 0.4141, 0.6169$ $\frac{1}{12}\{0 + 4(0.0600 + 0.4141) + 2 \times 0.2150 + 0.6169\}$ Hence required approximation is 0.245</p>	x	$\tan^{-1} x$	$2 \tan^{-1}(x-1)$	1.535	0.993	0.983	1.545	0.996	0.998	<p>B1</p> <p>M1 A1√</p> <p>B1 B1</p> <p>B1</p> <p>M1 A1</p> <p>M1 M1 A1</p>	<p>1 For correct exact value stated</p> <hr/> <p>2 For correct answer, following their a</p> <hr/> <p>3 For correct statement of asymptotes</p> <hr/> <p>2 For correct details and explanation</p> <hr/> <p>3 For the relevant function values seen or implied; must be radians, not degrees</p> <p>For use of correct formula with $h = \frac{1}{4}$</p> <p>3 For correct (2 or 3sf) answer</p> <p style="text-align: center;">4</p>
x	$\tan^{-1} x$	$2 \tan^{-1}(x-1)$									
1.535	0.993	0.983									
1.545	0.996	0.998									